

Erosion in the vicinity of the front point of a blunt body over which a dusty hypersonic stream flows is investigated. A three-velocity model of the motion of a multiphase medium is used to describe the process. The coefficient of erosion as a function of the parameters of the oncoming stream is determined as a result. The so-called screening effect, earlier studied experimentally in [1], is explained by particle deceleration in the dense dust layer above the eroded surface. It is shown that the steady character of the flow in the vicinity of the critical point is disrupted when dust is present if the mass concentration of particles in the undisturbed stream exceeds a certain critical value.

A number of papers [2, 3] have been devoted to the investigation of particle motion in hypersonic shock layers, but without allowance for the influence of erosion products. Erosion in the vicinity of the critical point of a subsonic, slightly dusty stream was investigated in [4]; the dependence of the erosion volume on the parameters of particle collision with the wall was assumed to be known and was borrowed from experimental research. The calculated model of erosive destruction suggested in [5] was used in the present work.

1. Two-Layer Flow of a Mixture in the Vicinity of the Critical Point. Let us consider a compressed layer on the flat face of a cylinder in a hypersonic equilibrium stream of a monodisperse mixture of gas and solid particles. The velocity vector of the oncoming stream is parallel to the axis of the cylinder, so that the flow is axisymmetric. We set the origin of coordinates at the center of the face, denoting the axial coordinates as x and the radial one as y (Fig. 1). We take the shape of the shock wave in the vicinity of the critical point as roughly parabolic,

$$x_s = s - y^2/2R_c^2,$$

R_c and s being the radius of curvature and the standoff distance of the shock wave, respectively.

Inert particles entering the compressed layer at hypersonic velocity are slowed only slightly and reach the surface of the cylinder having a large store of kinetic energy. Destruction of the surface of the body in the stream (erosion) occurs as a result of the numerous impacts of high-velocity particles. In each act of collision a certain volume of material of particles and body is ejected into the stream, and a dust layer forms above the eroded surface. We shall consider the dust layer to be plane and thin. If Δ is the thickness of the dust layer, then

$$\Delta \ll s. \quad (1.1)$$

We divide the region of flow in two: the region of relaxation of the monodisperse mixture, D_1 ($\Delta \ll x \ll x_s$, $y \geq 0$), where a two-velocity model is applicable, and the region of flow of the polydisperse mixture, D_2 ($0 \leq x \leq \Delta$, $y \geq 0$), in which we shall describe the flow within the framework of a three-velocity, quasi-one-dimensional model. We impose the restriction

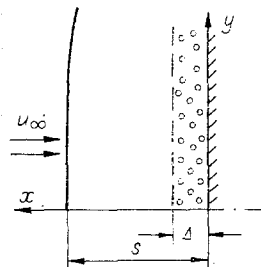


Fig. 1

$$\rho_p \ll \rho_\infty, \rho_e \quad (1.2)$$

on the parameters of the particle flux, where ρ_p is the density of the particle flux; ρ_∞ is the gas density in the oncoming stream; ρ_e is the flux density of erosion products. By virtue of (1.2), the equations of gas dynamics can be integrated in region D_1 independently of the equations of particle motion. At the boundary of the dust layer ($x = \Delta$) we demand continuity of all the parameters of the gas stream. Then in region D_1 the solution of the dynamic equations for an inviscid gas has the usual form for flow in the vicinity of a critical point (e.g., see [6]),

$$\rho = \rho_c, \quad u = -2F(x_1), \quad v = yF'(x_1), \quad F'(0) = k_c, \quad x_1 = x - \Delta(1 - \eta), \quad (1.3)$$

where ρ_c is the gas density beyond the straight section of the shock wave; k_c is the velocity gradient in the vicinity of the critical point; η is a constant of integration, which we find from the condition

$$\pi y^2 \rho u(\Delta) + 2\pi y \int_0^\Delta \rho v dx = 0.$$

Using (1.1) and (1.3), we can write this condition in the form

$$\eta = \frac{1}{k_c y \Delta} \int_0^\Delta v dx.$$

Thus, the quantity η is defined as the average, over the thickness of the dust layer, radial velocity component of the gas stream.

The flow of the gas-dust mixture in region D_2 will be described within the framework of the Kligel-Nickerson model [7]. We take the mixture to be in temperature equilibrium, while we characterize the dynamics of all the erosion fragments by one relaxation parameter τ . The influence of particles on the parameters of the flux of erosion products in the volume of the flow can be neglected by virtue of (1.2). The model equations averaged over the thickness of the dust layer have the form (we omit the averaging parentheses)

$$\begin{aligned} \frac{\rho u(\Delta)}{\Delta} + \frac{1}{y} \frac{\partial}{\partial y} y \rho v &= 0, \\ \rho \frac{u(\Delta)v(\Delta)}{\Delta} + \frac{1}{y} \frac{\partial}{\partial y} y \rho v^2 + \frac{\partial p}{\partial y} + \frac{\rho_e}{\tau} (v - v_e) &= 0, \\ \frac{\Delta}{y} \frac{\partial}{\partial y} y \rho_e v_e &= J_e, \quad \frac{1}{y} \frac{\partial}{\partial y} y \rho_e v_e^2 = \frac{\rho_e}{\tau} (v - v_e), \end{aligned} \quad (1.4)$$

where $u(\Delta)$ and $v(\Delta)$ are the velocity components of the gas stream at the boundary of the dust layer; J_e is the mass flux of erosion products from the face surface.

Equations (1.4) describe the transfer in the radial direction of the mass and momentum of components of the mixture. Parameters of the flux of the solid phase are marked by the index e . The boundary conditions for the system (1.4) at $y = 0$ are formulated in the form

$$v = v_e = 0, \quad \partial \rho / \partial y = \partial \rho_e / \partial y = 0. \quad (1.5)$$

Since the gas in the shock layer is assumed to be incompressible, the radial pressure gradient in the dust layer will be the same as at the boundary $x = \Delta$. Thus,

$$\frac{\partial p}{\partial y} = -\rho k_c^2 y.$$

We assume that $J_e(y) = \text{const}$, and then the solution of the problem (1.4), (1.5) can be represented in the form

$$\rho = \rho_c, \quad v = \eta k_c y, \quad \rho_e = \text{const}, \quad v_e = \xi k_c y. \quad (1.6)$$

Substituting (1.6) into (1.4), we obtain the system of algebraic equations

$$3\eta^2 - 2\eta + \frac{\rho_e}{\rho} \frac{2}{q} (\eta - \xi) = 1, \quad 3\xi^2 = \frac{2}{q} (\eta - \xi), \quad \rho_e = \frac{J_e}{\xi 2k_c \Delta}, \quad (1.7)$$

where $q = 2k_c \tau$. The expression (1.6), together with the system (1.7), allow us to determine the averaged parameters of the dust layer if J_e and Δ are known.

By virtue of the law of conservation of mass, the flux of erosion products is connected with the particle flux to the eroded surface by the relation

$$J_e = -(1 + E)(\rho_p u_p)_{x=0}, \quad (1.8)$$

where E is the coefficient of erosion. To determine the thickness of the dust layer, we integrate the equations of motion of the erosion fragment along the axial line

$$u_e \frac{\partial u_e}{\partial x} = \frac{1}{\tau} (u - u_e),$$

at which we set $u = -2\eta k_e x$. As a result, for $\eta q < 1/4$ we have

$$u_e^2 + \frac{x}{\tau} u_e + \eta q \left(\frac{x}{\tau} \right)^2 = u_e^2(0) \left| \frac{\tau u_e - z_1 x}{\tau u_e - z_2 x} \right|^\alpha,$$

where $z_1 = -1/2 + \sqrt{1/4 - \eta q}$; $z_2 = -1/2 - \sqrt{1/4 - \eta q}$; $\alpha = (1 - 4\eta q)^{-1/2}$; $u_e(0)$ is the velocity of escape of erosion fragments from the surface. At the outer boundary of the layer ($x = \Delta$) we must have $u_e(\Delta) = 0$, from which we find

$$\Delta = \tau u_e(0) \left(\frac{1}{\eta q} \left| \frac{z_1}{z_2} \right|^\alpha \right)^{1/2}.$$

For small values of ηq we have

$$\Delta = \tau u_e(0)(1 + \eta q \ln \eta q), \quad \eta q \ll 1. \quad (1.9)$$

Analyzing this expression, we can conclude that the influence of the counterflow of gas on the thickness of the dust layer is insignificant when the interaction parameter ηq is small.

The initial velocity of the erosion fragments is determined by the velocity of collision of particles with the surface, $u_e(0) = -\lambda u_p(0)$, λ being the coefficient of restitution of velocity upon impact. Neglecting the second term inside the parentheses in (1.9), which is small compared with unity, we finally obtain

$$\Delta \approx -\tau \lambda u_p(0), \quad \eta q \ll 1. \quad (1.10)$$

Thus, the parameters of the dust layer are determined from the known conditions of collision of the particles with the wall. The parameters of the particle flux are determined, in turn, by the conditions of motion in the dust layer.

2. Particle Motion in a Dusty Hypersonic Shock Layer. Particles striking a shock layer from an undisturbed stream pass successively through the region D_1 of pure gas and the region D_2 of dusty gas. In this case the drag force acting on a particle will depend both on the interaction with the viscous gas and collisions with erosion fragments. In the general case, the equations describing the flow of a monodisperse solid phase in a mixture have the form [8]

$$\begin{aligned} \frac{\partial}{\partial x} \gamma \rho_p u_p + \frac{\partial}{\partial y} \gamma \rho_p v_p &= 0, \\ \left(u_p \frac{\partial}{\partial x} + v_p \frac{\partial}{\partial y} \right) \mathbf{v}_p &= \frac{3}{8} \frac{\rho C_D}{\rho_s r_p} |\mathbf{v} - \mathbf{v}_p| (\mathbf{v} - \mathbf{v}_p) + \frac{3}{8} \frac{\rho_e C'_D}{\rho_s r_p} |\mathbf{v}_e - \mathbf{v}_p| (\mathbf{v}_e - \mathbf{v}_p), \end{aligned} \quad (2.1)$$

where ρ_s and r_p are the density and radius of a particle, respectively; C_D is the drag coefficient in the gas stream; C'_D is the drag coefficient in the stream of erosion products. The following conditions are satisfied on the shock-wave line and on the line of symmetry:

$$\begin{aligned} x = x_s, \quad \rho_p &= \rho_{p\infty}, \quad u_p = u_\infty, \quad v_p = 0, \\ y = 0, \quad \partial \rho_p / \partial y &= 0, \quad \partial u_p / \partial y = 0, \quad v_p = 0. \end{aligned} \quad (2.2)$$

When investigating the erosion of a body in a dusty hypersonic stream, we can simplify the problem (2.1), (2.2) considerably. In this case we assume [2] that the following conditions are satisfied everywhere in the compressed layer:

$$|u_p| \gg v_p, \quad |\mathbf{v}|, \quad |\mathbf{v}_e|.$$

Moreover, for $y^2 \ll R_c^2$ we can neglect the curvature of the shock front. Then we set $|\mathbf{v} - \mathbf{v}_p| \approx |u_p|$, $|\mathbf{v}_e - \mathbf{v}_p| \approx |u_p|$ in (2.1), while under the conditions (2.2) we can take $x_s = s$. As a result, we obtain the system of equations

$$\begin{aligned} \frac{\partial}{\partial x} y \rho_p u_p + \frac{\partial}{\partial y} y \rho_p v_p &= 0, \\ \left(u_p \frac{\partial}{\partial x} + v_p \frac{\partial}{\partial y} \right) u_p &= \left(l_p^{-1} + \frac{3}{8} \frac{\rho_e C_D'}{\rho_s r_p} \right) |u_p| u_p, \quad \left(u_p \frac{\partial}{\partial x} + v_p \frac{\partial}{\partial y} \right) v_p = \\ &= l_p^{-1} |u_p| (v - v_p) + \frac{3}{8} \frac{\rho_e C_D'}{\rho_s r_p} |u_p| (v_e - v_p), \end{aligned} \quad (2.3)$$

where $l_p = (8/3C_D)(\rho_s/\rho)r_p$. We shall solve the system (2.3) under the conditions

$$\begin{aligned} x = s, \quad \rho_p &= \rho_{p\infty}, \quad u_p = u_\infty, \quad v_p = 0, \\ y = 0, \quad \partial \rho_p / \partial y &= 0, \quad \partial u_p / \partial y = 0, \quad v_p = 0. \end{aligned} \quad (2.4)$$

Further, we take conditions of particle motion such that $C_D = \text{const}$ and $C_D' = \text{const}$. In region D_1 the particles move through pure gas, so that we must set $\rho_e = 0$ and $\mathbf{v} = y\mathbf{F}'(x_1)$ in (2.3). We seek the solution of the problem (2.3), (2.4) in region D_1 in the form

$$\rho_p = \rho_p(x), \quad u_p = u_p(x), \quad v_p = k_p(x)y.$$

As a result, we have

$$\begin{aligned} \rho_p u_p &= \rho_{p\infty} u_\infty \exp \left[-2 \int_x^s \frac{F'(x_1)}{|u_p|} dx \right], \\ u_p &= u_\infty \exp \left(-\frac{s-x}{l_p} \right), \\ u_p \frac{dk_p}{dx} + k_p^2 &= \frac{|u_p|}{l_p} [F'(x_1) - k_p], \quad k_p(s) = 0. \end{aligned}$$

Let us estimate the degree of scattering of the particle flux in the radial direction. Obviously, $k_p < k_c$, from which we have the inequality

$$\frac{(\rho_p u_p)_{x=\Delta}}{\rho_{p\infty} u_\infty} < \exp \left[-\frac{2k_c l_p}{|u_\infty|} \left(e^{\frac{s-\Delta}{l_p}} - 1 \right) \right] \approx e^{-\varepsilon A},$$

where $A = \frac{l_p}{s} \left(e^{\frac{s-\Delta}{l_p}} - 1 \right)$; $\varepsilon = \rho_\infty / \rho_c$. Since $\varepsilon \ll 1$ for hypersonic conditions of streamline flow, the particle flux will be scattered significantly only for $A \sim \varepsilon^{-1}$. By virtue of the same limitations under which the problem is being solved, we must require that the inequality $l_p > s$ be satisfied and neglect scattering of the flux.

The problem is solved similarly in region D_2 . Integrating the second equation in the system (2.3), we obtain an expression for the axial component of the particle velocity at the face surface,

$$u_p(0) = u_\infty \exp \left(-\frac{s}{l_p} - \Phi \right); \quad \Phi = \frac{3}{8} \frac{C_D'}{\rho_s r_p} \int_0^\Delta \rho_e dx.$$

The estimate of the degree of scattering of the particle flux in the dust layer has the form

$$\frac{(\rho_p u_p)_{x=0}}{\rho_{p\infty} u_\infty} < \exp \left[-\varepsilon \frac{l_p}{s} \left(e^{-s/l_p} - 1 \right) - \frac{\rho_{p\infty}}{\langle \rho_e \rangle} e^\Phi \right],$$

where $\langle \rho_e \rangle$ is the average density of erosion products in the dust layer. With allowance for the condition (1.2), we can neglect scattering of the particle flux when $\Phi \lesssim 1$. Summarizing all this, we write the equations connecting the parameters of the particle flux at the wall with the parameters characterizing the dust layer:

$$\begin{aligned} (\rho_p u_p)_{x=0} &\approx \rho_{p\infty} u_\infty, \quad s < l_p, \quad \Phi \lesssim 1, \\ u_p(0) &= u_\infty \exp \left(-\frac{s}{l_p} - \Phi \right), \quad \Phi = \frac{3C_D'}{8} \frac{\langle \rho_e \rangle \Delta}{\rho_s r_p}. \end{aligned} \quad (2.5)$$

The parameter Φ determines the degree of screening of the surface by the layer of erosion fragments. Thus, the screening effect is explained by particle deceleration in the dense dust layer. Another mechanism capable of resulting in surface screening is the fragmentation of particles in collisions with erosion fragments. This kind of effect was studied in [9]. It

is observed under the conditions of rain erosion, where drop fragmentation actually is an important fact.

3. Determination of the Coefficient of Erosion. At a high velocity of particle collision with the surface, the dependence of the coefficient of erosion on the particle velocity has the form [6]

$$E = u_p^2 / 2H_{en}$$

H_{en} being the effective enthalpy of erosional destruction. With allowance for (2.5), we can write this expression in the form

$$E = E_0 e^{-2\Phi}, \quad E_0 = \frac{u_\infty^2}{2H_{en}} e^{-2s/l_p}. \quad (3.1)$$

Further, we assume that in the investigated range of collisional velocities the coefficient of restriction of velocity during an impact is constant, $\lambda = \text{const}$. Then from (1.10) and (2.5) we get

$$\Delta = \Delta_0 e^{-\Phi}, \quad \Delta_0 = \lambda \tau |u_\infty|. \quad (3.2)$$

Substituting (3.1) and (3.2) into the system (1.7), we have

$$3\eta^2 - 2\eta + 3\xi^2 a \Phi e^\Phi = 1, \quad \eta = \xi + (3/2)q\xi^2, \quad (1 + E_0 e^{-2\Phi})p = \xi\Phi, \quad (3.3)$$

where

$$p = \frac{3C'_D \rho_{p\infty} |u_\infty|}{8 \rho_s r_p^2 k_c}; \quad a = \frac{8 \rho_s r_p}{3C'_D \rho \Delta_0}.$$

Since $\eta q \ll 1$ and $\xi < \eta$, $\eta = \xi + O(\eta^2 q)$. Therefore, from (3.3) we get

$$\xi = \frac{1 + \sqrt{1 + 3(1 + a\Phi e^\Phi)}}{3(1 + a\Phi e^\Phi)} + O(\eta^2 q).$$

From this we obtain the equation connecting the screening parameter with the parameters of the oncoming stream:

$$p = \frac{\Phi}{1 + E_0 e^{-2\Phi}} \frac{1 + \sqrt{1 + 3(1 + a\Phi e^\Phi)}}{3(1 + a\Phi e^\Phi)}. \quad (3.4)$$

For $E_0 = 0$ the function $p(\Phi)$ has the form of a hump with the maximum at the point $\Phi = \Phi_m$:

$$\Phi_m = 1 + \frac{2e^{-1}}{a} + O(a^{-2}), \quad a \gg 1.$$

Here $p(0) = 0$ and $p(\Phi) \rightarrow 0$, when $\Phi \rightarrow \infty$. Hence it follows that the function $\Phi(p)$ has two branches and is defined only in the interval $0 \leq p \leq p^*$, $p^* = p(\Phi_m)$. The physical branch of the solution is chosen from the condition $\Phi(0) = 0$. Finite values of E_0 do not alter the character of the behavior of the function $p(\Phi)$ but only alter the position of the extremal point. Thus, the steady-state solutions indicated above, describing flow in the vicinity of a critical point in the presence of a dust layer, do not exist if the particle concentration in the undisturbed stream exceeds the critical value, i.e., if $p > p^*$.

Combining (3.1) and (3.4), we find the functional relation $E = E(p, \alpha, E_0)$. In Fig. 2a we present calculated relations $E = E(p)$ for $\alpha = 10, 20$, and 30 (lines 1-3, respectively) for $E_0 = 1$. The influence of the initial value of the erosion coefficient can be traced in the data of Fig. 2b, where the relations $E = E(p)$ satisfied for $\alpha = 20$ and $E_0 = 0.1, 1.0$, and 10 (lines 1-3, respectively) are calculated. The qualitative agreement with experimental results [1] can be noted, although a direct quantitative comparison is impossible since the data in [1] were treated using a criterial relation $E = E(\rho_p u_{p\infty}, \rho_c, r_p)$. Each of the curves in Fig. 2a, b is determined up to a critical value $p = p^*(\alpha, E_0)$. Analyzing these data, we can conclude that the screening effect is intensified with an increase in p, α , and E_0 .

The parameters E and E_0 depend differently on the velocity of the oncoming stream. Since the ratio s/l_p depends little on u_∞ (hypersonic stabilization), in the general case we set $E_0 \sim u_\infty^n$ and estimate the exponent in the law $E \sim u_\infty^m$. From Eq. (3.1) we find

$$m = n(1 - \partial\Phi/\partial \ln u_\infty).$$

Typical curves of the relation $m = m(\Phi)$ are presented in Fig. 3. The calculations were made for $n = 2, \alpha = 20$, and $E = 0.1, 1.0$, and 10 (lines 1-3, respectively). In analyzing these

data, we note the following: 1) In the investigation of erosion-resistant materials, experiments with dusty streams yield an overstated value of the exponent in the law $E \sim u_{\infty}^m$; 2) in the investigation of materials with a low erosion resistance ($E_0 \gg 1$) the analogous exponent will be lower than the standard value for low stream dustiness ($p \ll p^*$) and higher than the standard value for high dustiness ($p \sim p^*$). The exponent in the law $E_0 \sim u_{\infty}^n$ determined in experiments on the collision of single particles is taken as the standard value.

Let us determine the influence of stream dustiness on the forces acting on the body in streamline flow on the part of the particle flux. With allowance for the momentum flux of erosion fragments leaving the surface, we have

$$F \sim (\rho_p u_p^2) [1 + \lambda(1 + E)],$$

F being the effective force. Using the results obtained, we find

$$F/F_0 = p[1 + \lambda(1 + E_0 e^{-2\Phi})] e^{-\Phi},$$

where $F_0 = \frac{C_D}{C'_D} \frac{l_p}{s} e^{-s/l_p} \rho_{\infty} u_{\infty}^2$.

Under erosion conditions a collision is almost inelastic ($\lambda \ll 1$), so that for $E_0 \leq 1$ we have

$$F/F_0 = p \exp(-\Phi(p)). \tag{3.5}$$

Thus, the effective forces can be lower than for inelastic impact.

For the criteria p and a we have, in accordance with their definitions, the following equivalent expressions:

$$p = \frac{C'_D \rho_{p\infty} s}{C_D \rho_{\infty} l_p}, \quad a = \frac{C_D l_p}{C'_D \Delta_0}.$$

The estimate for the critical value of p has the form

$$p^* \approx (3ae)^{-1/2} / (1 + E_0 e^{-2}).$$

From the condition (1.1) we get the inequality $a \gg 1$, from which $p^* \ll 1$. Therefore, in the flow of a dusty hypersonic stream over a blunt body the dust layer formed in the bow part of the body can significantly affect the parameters of the erosion process even for a low dustiness of the oncoming stream.

Finally, we note that the constant characterizing the resistance of the material to erosive destruction, i.e., H_{en} , actually depends on the temperature T_w of the surface being eroded. In the process of destruction a certain share of the kinetic energy of the particles is expended on heating the surface, so that $T_w = T_w(\dot{\epsilon}_p)$, where $\dot{\epsilon}_p$ is the flux of kinetic energy of the particles. From this we get the functional relation $H_{en} = H_{en}(\dot{\epsilon}_p)$, which must be allowed for in the calculations. According to the results of [10], however, the temperature dependence of the coefficient of erosion is manifested only near a certain temperature, specific for the given material, close to the melting temperature T_{melt} . But if the surface temperature is considerably lower than the melting temperature, one can take $H_{en}(T_w) \approx \text{const}$.

Thus, the conclusions following from an analysis of Eq. (3.4) correspond best to the linear stage of erosion of refractory materials such as graphite, tungsten, etc. An additional restriction can be written in the form of the inequality $dH_{en}/dT_w \ll H_{en}/T_w$, defining the region of the parameters where the enthalpy of erosive destruction actually is constant.

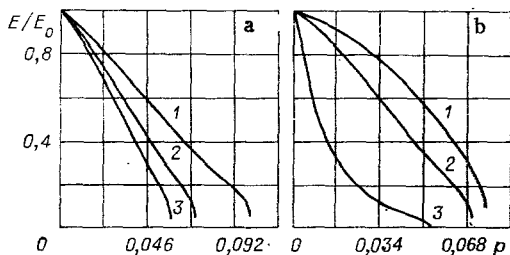


Fig. 2

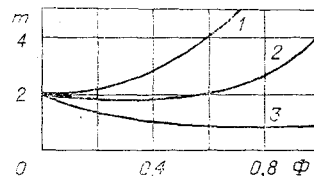


Fig. 3

LITERATURE CITED

1. A. J. Laderman, C. H. Lewis, and S. R. Byron, "Two-phase plume impingement effects," AIAA J., 8, No. 10 (1970).
2. R. F. Probstein and H. Fassio, "Dusty hypersonic flows," AIAA J., 8, No. 4 (1970).
3. A. P. Vasil'kov, "Vicinity of the critical point of a blunt body in a hypersonic two-phase stream," Izv. Akad. Nauk SSSR, Mekh. **Zhidk. Gaza**, No. 5 (1975).
4. J. A. Latone, "Erosion prediction near a stagnation point resulting from aerodynamically entrained solid particles," J. Aircraft, 16, No. 12 (1979).
5. Yu. V. Polezhaev, V. P. Romanchenkov, et al., "Calculating model of the process of erosional destruction of a composite material," Inzh.-Fiz. Zh., 37, No. 3 (1979).
6. N. F. Krasnov, Aerodynamics of Bodies of Revolution [in Russian], Mashinostroenie, Moscow (1964).
7. J. Kligel and G. Nickerson, "Flow of a mixture of gas and solid particles in an axisymmetric nozzle," in: Detonation and Two-Phase Flow [Russian translation], Mir, Moscow (1966).
8. L. E. Sternin, Principles of the Gas Dynamics of Two-Phase Flows in Nozzles [in Russian], Mashinostroenie, Moscow (1974).
9. W. G. Reinecke, "Debris shielding during high-speed erosion," AIAA J., 12, No. 11 (1974).
10. T. Wakeman and W. Tabakoff, "Erosion behavior in a simulated jet engine environment," J. Aircraft, 16, No. 12 (1979).